S-duality

~~Fiber-Base duality~~

and

Global Symmetry Enhancement

Futoshi Yagi (KIAS)

Based on arXiv: 1411.2450

V. Mitev, E.Pomoni, M.Taki, FY

5D $\mathcal{N}=1$ SUSY SU(2) gauge theory with N_f flavor

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'96 Seiberg

UV fixed point exists for $N_f \leqq 7$

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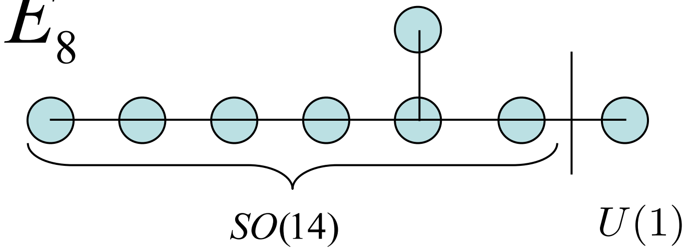
Global symmetry enhancement at UV fixed point

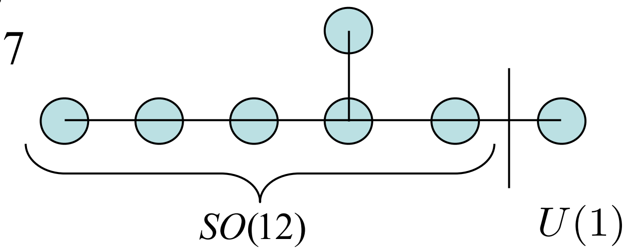
$$SO(2N_f) \times U(1) \subset E_{N_f+1}$$

N_f flavors ↗ ; Instanton particle ↖

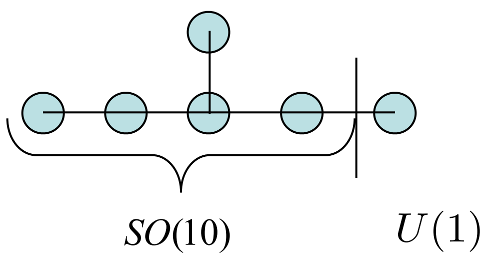
E_8



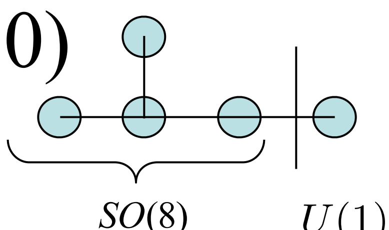
E_7



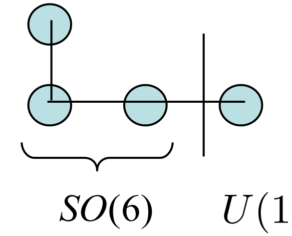
E_6



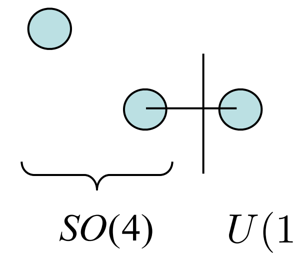
$E_5 = SO(10)$



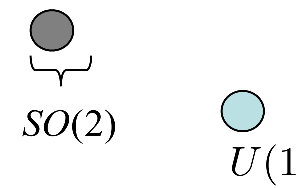
$E_4 = SU(5)$



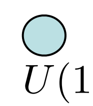
$E_3 = SU(2) \times SU(3)$



$E_2 = U(1) \times SU(2)$



$E_1 = SU(2)$



5D *N=1* SUSY SU(2) gauge theory with N_f flavor

'96 Seiberg

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N_f flavors ↗ ↖ Instanton particle

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→ **Prepotential is invariant**

→ **Low energy effective action at Coulomb phase is invariant**

How about 5D Nekrasov partition function?

$$Z^{5D}_{Nek}(q, m, a; \epsilon_1, \epsilon_2)$$

$q = e^{-\frac{\beta}{2g^2}}$: instanton factor

(g : gauge coupling β : circumference)

m : mass parameter

a : Coulomb moduli parameter

ϵ_1, ϵ_2 : Ω deformation parameter

- Defined for 5D N=1 theory compactified on S^1
- Prepotential is reproduced by $\epsilon_1, \epsilon_2 \to 0$

Is 5D Nekrasov partition function invariant under the Weyl transformation of E_{N_f+1} ?

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Paradox?

- **Does not look invariant…**

e.g. For pure SYM

$E_1 = SU(2): \quad q \leftrightarrow q^{-1}$

Instanton factor

$$Z^{5D}{}_{Nek}(q^{-1}, a, \varepsilon_1, \varepsilon_2) \neq Z^{5D}{}_{Nek}(q, a, \varepsilon_1, \varepsilon_2)?$$

Positive power in q

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Resolution

5D Nekrasov partition function is invariant

$$Z^{5D}{}_{Nek}(q^{-1}, \underline{a'(a,q)}, \varepsilon_1, \varepsilon_2) = Z^{5D}{}_{Nek}(q, a, \varepsilon_1, \varepsilon_2)$$

Coulomb moduli parameter is also transformed under the E_{N_f+1} Weyl symmetry

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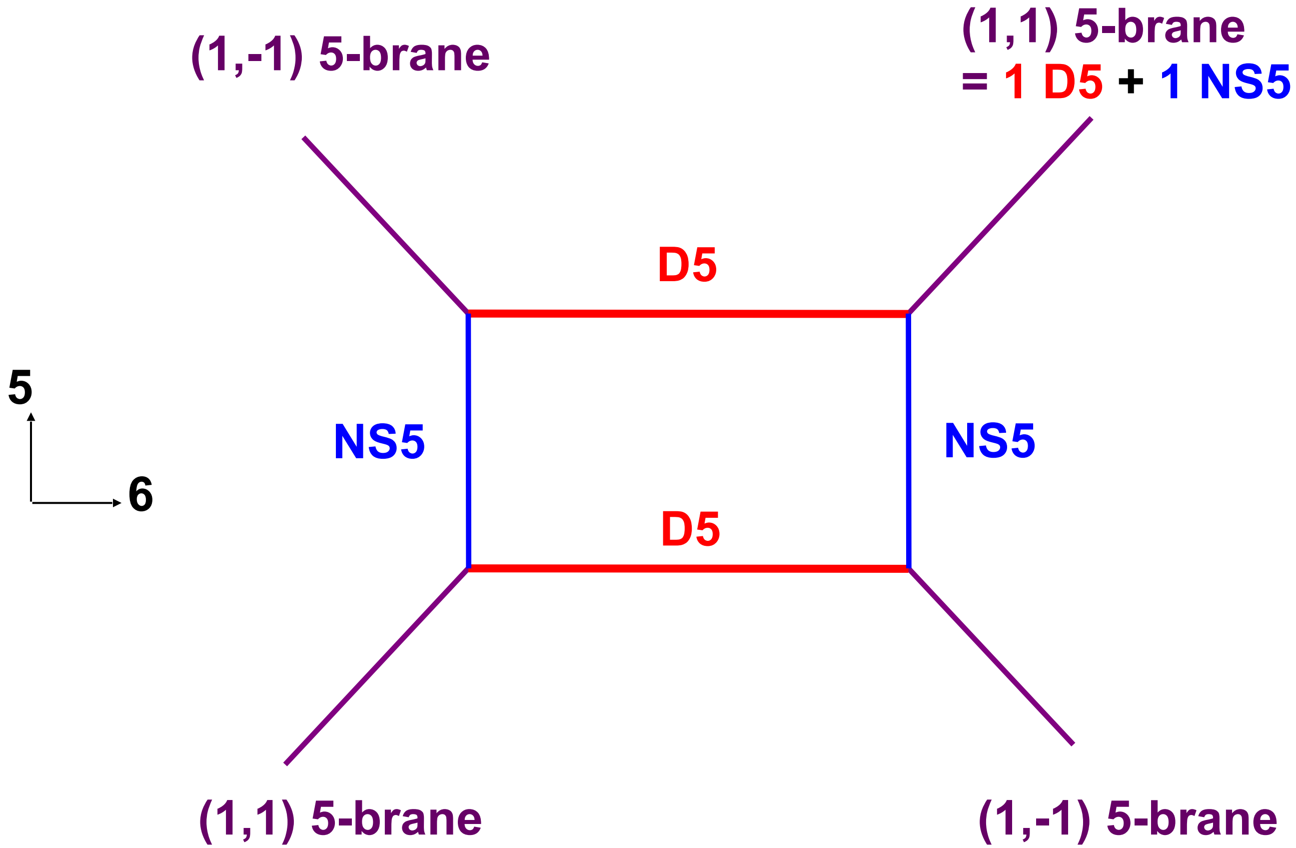
How to understand this?

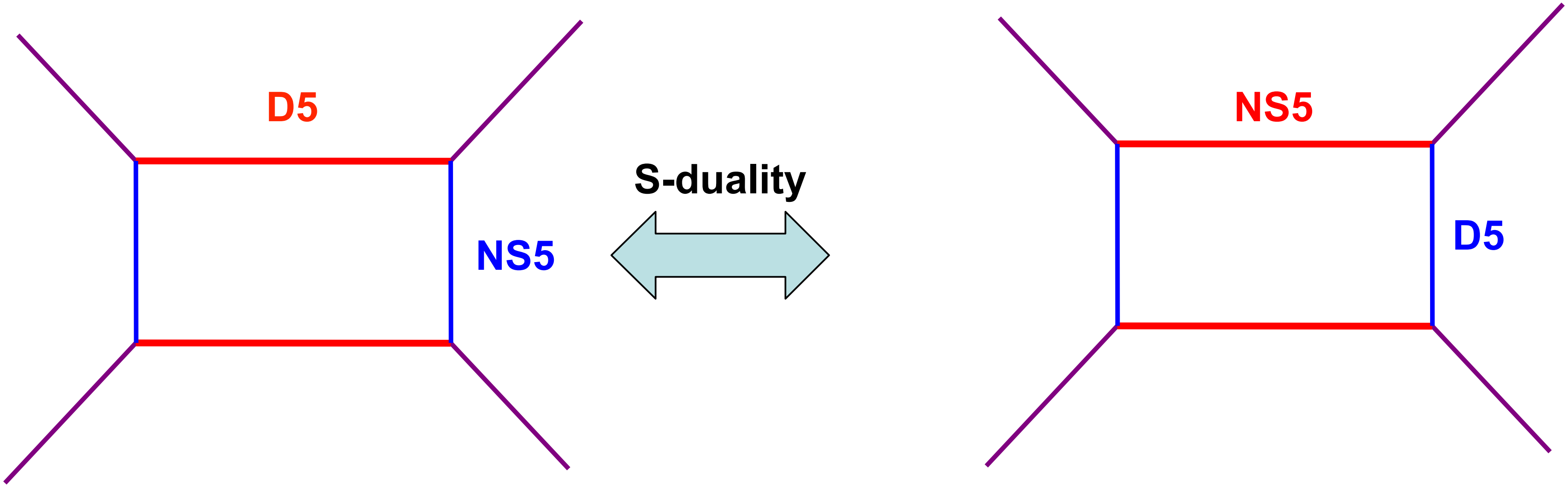
S-duality acting 5-branes

Brane setup for pure SU(2) SYM



NS5	0 1 2 3 4 5
D5	0 1 2 3 4 6

S-duality for pure SU(2) SYM



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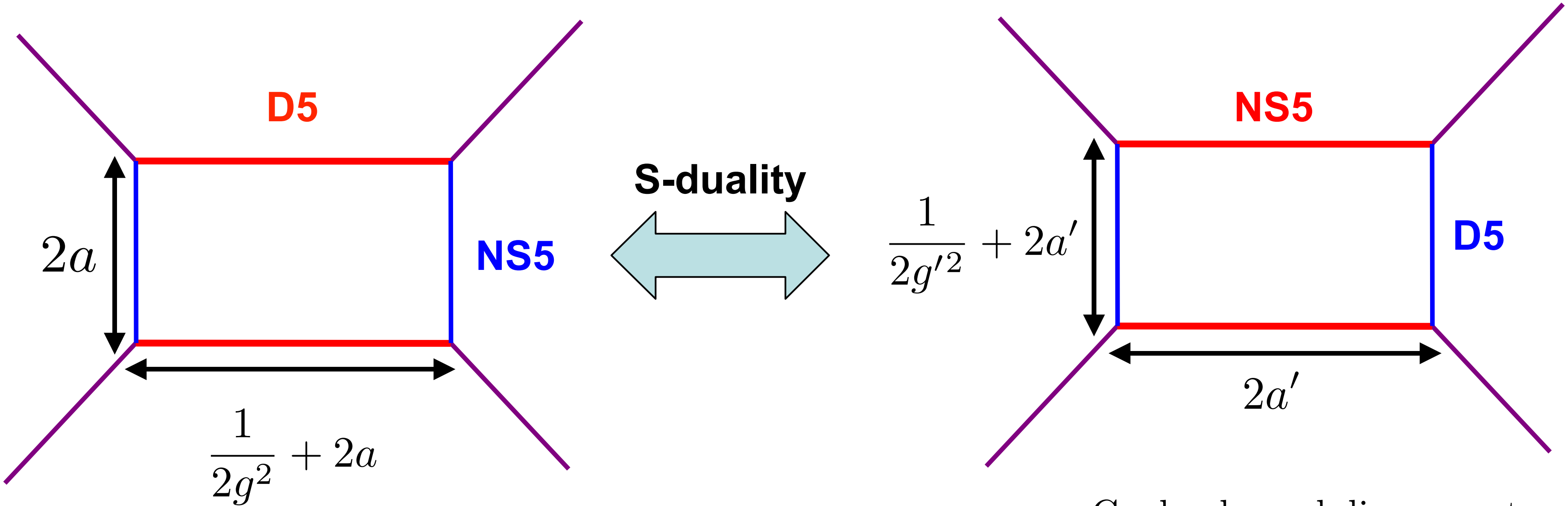
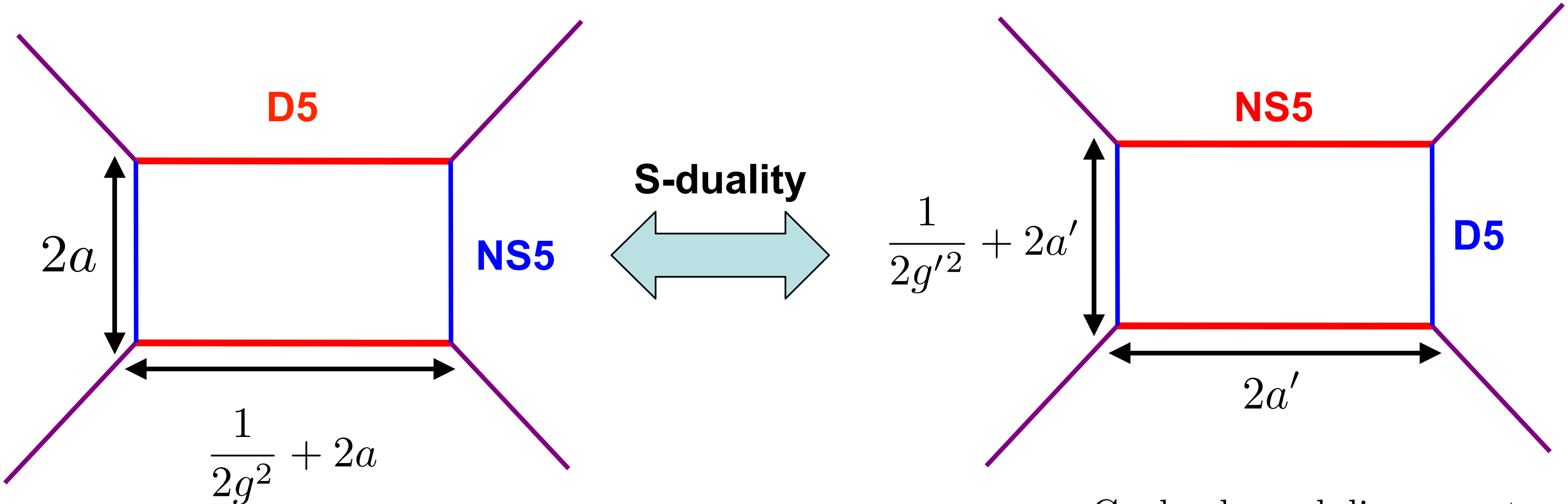


a : Coulomb moduli parameter

g : (Bare) gauge coupling

S-duality for pure SU(2) SYM



$$2a \leftrightarrow \frac{1}{2g^2} + 2a$$

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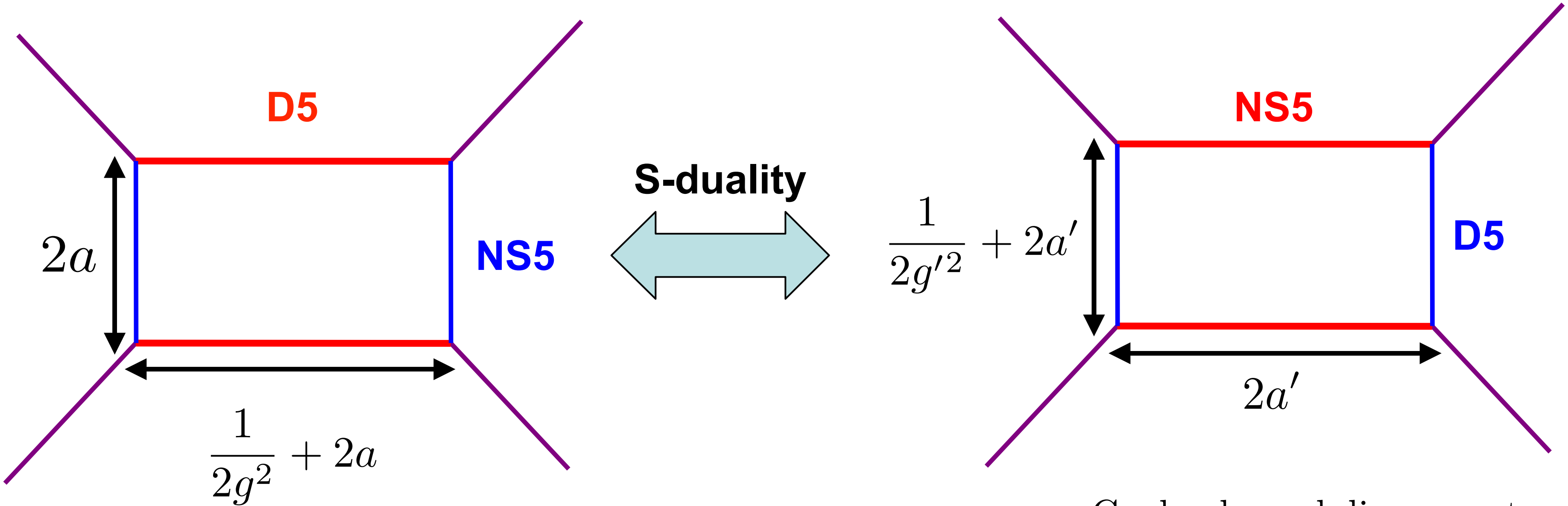

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$$a \to a + \frac{1}{4g^2}$$

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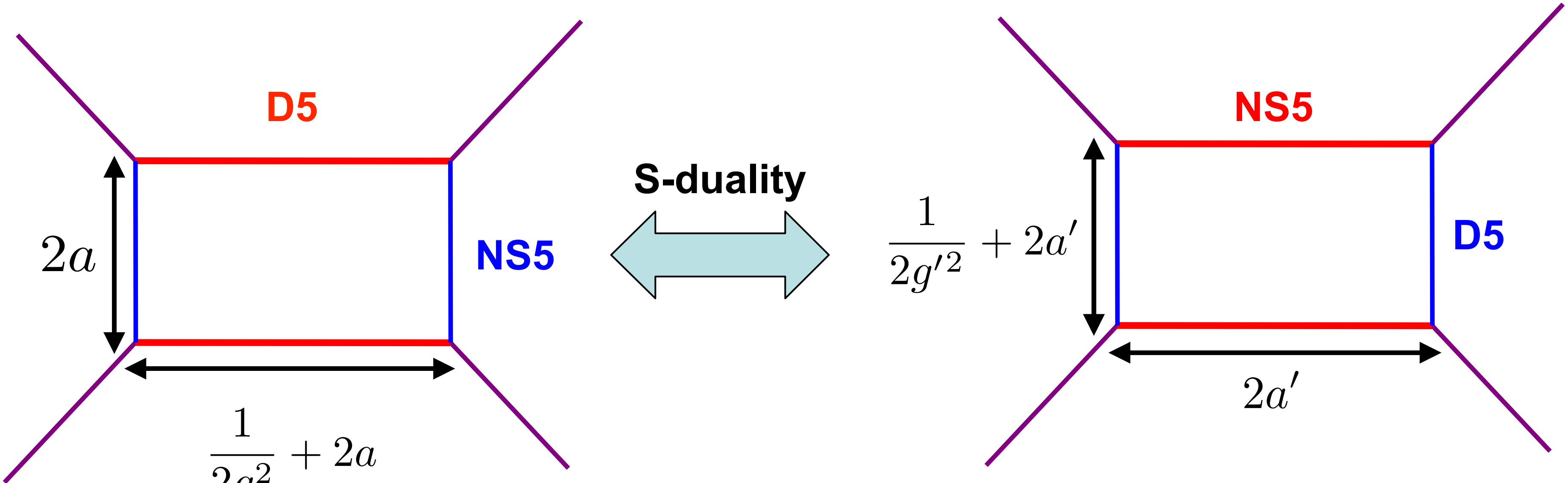

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Weyl Symmetry for $E_1 = SU(2)$

'97 Aharony,Hanany,Kol

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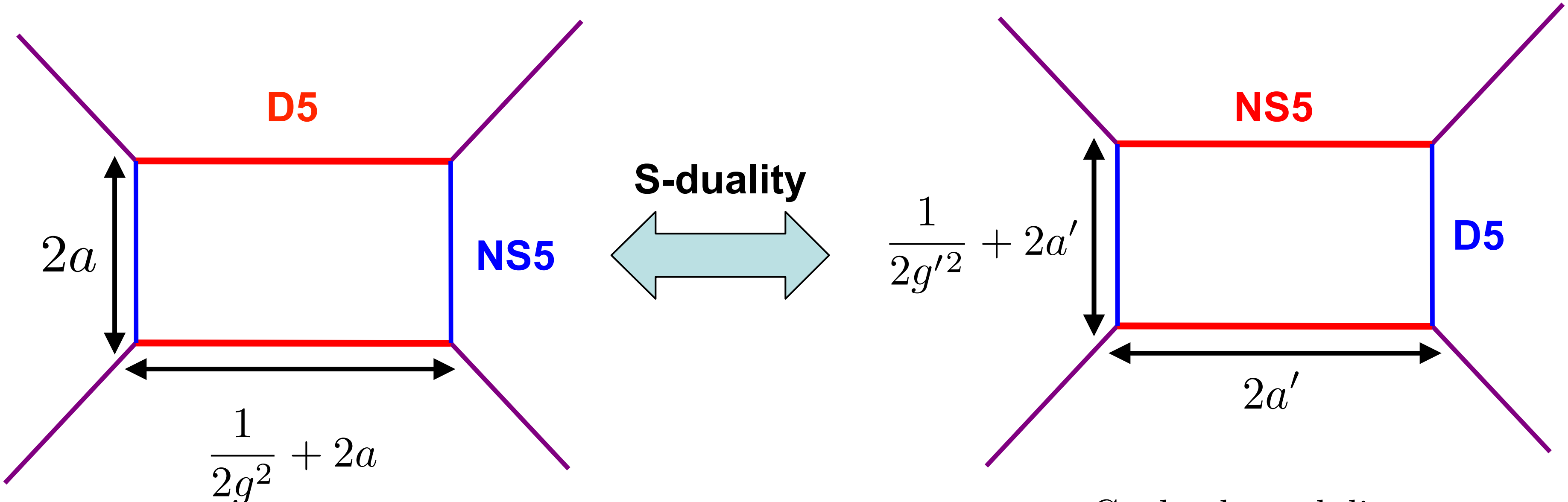

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Weyl Symmetry for E_1 = $SU(2)$

'97 Aharony,Hanany,Kol

$$a \to a + \frac{1}{4g^2}$$

Coulomb moduli parameter is also transformed!

We should use the new variable

$$\tilde{A} = q^{\frac{1}{4}} e^{\beta a} = e^{-\beta\left(a + \frac{1}{8g^2}\right)}$$

invariant under the E_1 Weyl transformation $(q \to q^{-1}, \quad a \to a + \frac{1}{2g^2})$
to rewrite Nekrasov partition function

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$$Z^{5D}{}_{Nek}(q, a; \epsilon_1, \epsilon_2) = Z_{pert}(a, \epsilon_1, \epsilon_2) \sum_{k=0}^{\infty} Z_k(a, \epsilon_1, \epsilon_2) q^k$$

Original form

’12 H-C Kim, S-S.Kim, K.Lee
’14 C.Hwang, J.Kim, S.Kim, J.Park

Talk by Chiung Hwang

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$$= \sum_{n=0}^{\infty} \underbrace{\tilde{Z}_n(q, \epsilon_1, \epsilon_2)}_{E_1 \text{ invariant}} \tilde{A}^n$$

E_1 invariant

New form

Talk by Joonho Kim

Nekrasov partition function for pure $SU(2)$

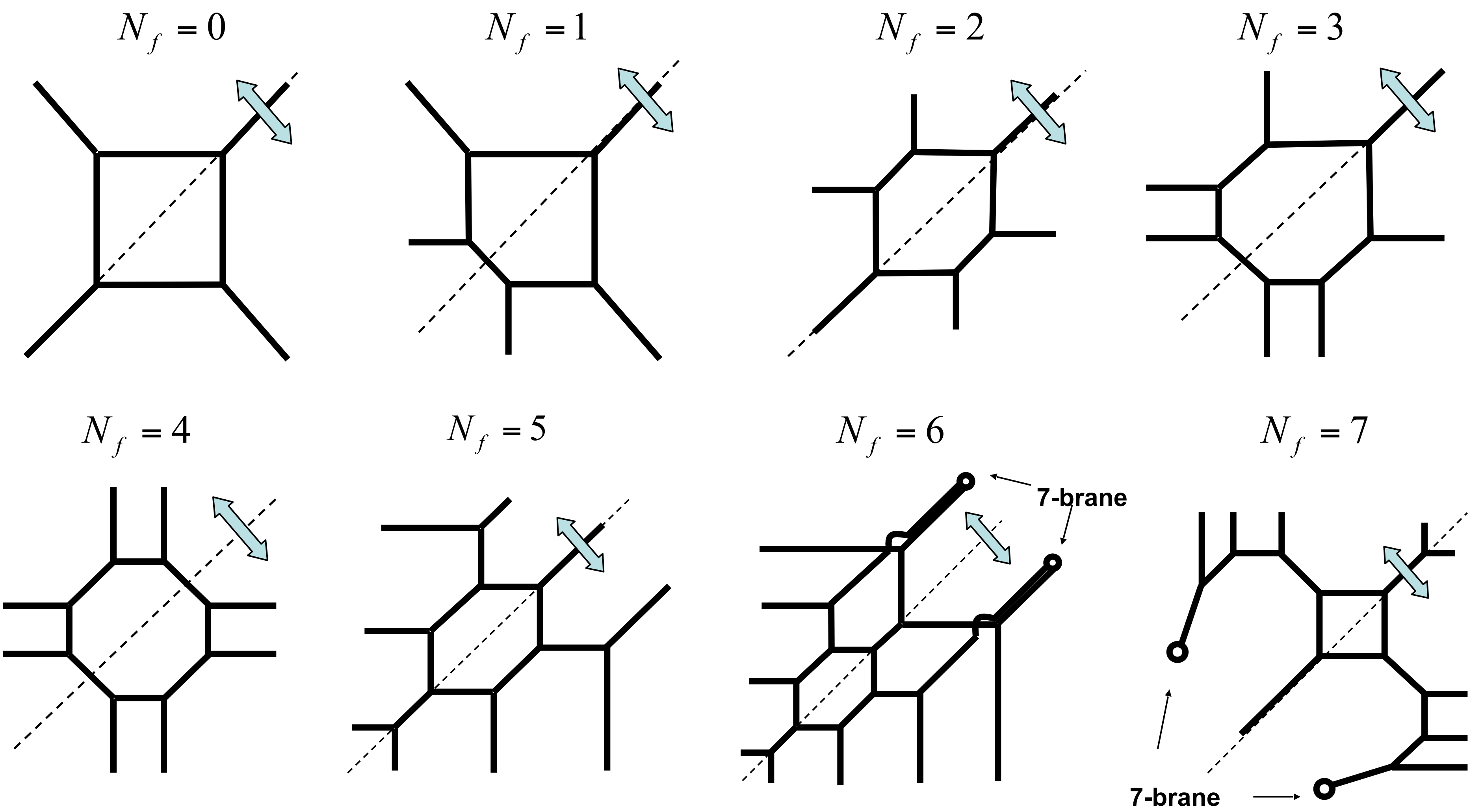
$$
\begin{aligned}
&Z^{5D}{}_{Nek}(q,a;\epsilon_1,\epsilon_2)\\
&= 1 + \frac{\mathfrak{t}+\mathfrak{q}}{(1-\mathfrak{t})(1-\mathfrak{q})}\chi_2^{E_1}(q)\tilde{A}^2\\
&\quad + \Bigg[\frac{(\mathfrak{q}^2+\mathfrak{t}^2)(\mathfrak{q}+\mathfrak{t}+\mathfrak{q}^2+\mathfrak{t}^2+\mathfrak{q}\mathfrak{t}(1+\mathfrak{q}+\mathfrak{t}))}{\mathfrak{q}\mathfrak{t}(1-\mathfrak{q})(1+\mathfrak{q})(1-\mathfrak{t})(1+\mathfrak{t})}\\
&\qquad + \frac{(\mathfrak{q}+\mathfrak{t}+\mathfrak{q}^2+\mathfrak{t}^2+\mathfrak{q}\mathfrak{t}(1+\mathfrak{q}+\mathfrak{t}))\chi_3^{E_1}(q)}{(1-\mathfrak{q})^2(1+\mathfrak{q})(1-\mathfrak{t})^2(1+\mathfrak{t})}\Bigg]\tilde{A}^4 + \mathcal{O}(\tilde{A}^6).
\end{aligned}
$$

Character of $E_1 = SU(2)$: $\chi_\ell(q) \equiv \sum_{m=0}^{2\ell} q^{m-2\ell}$

$\mathfrak{q} = e^{-\beta\epsilon_1}, \quad \mathfrak{t} = e^{\beta\epsilon_2}$

Manifestly E_1 invariant!!

Generalization to higher flavor



’09 Benini-Benvenuti-Tachikawa

(Weyl symmetry of)
$\mathbf{SO(2N_f) \times U(1)}$

+

Transformation induced from
S-duality

Flavors
(Masses)

Instanton particle
(Gauge coupling)

Flavors $\leftrightarrow$ Instanton particle
(Masses $\leftrightarrow$ Gauge coupling)

$\Rightarrow$

(Weyl symmetry of)
Enhanced $\mathbf{E_{N_f+1}}$ symmetry

We obtain manifestly E_{Nf+1} invariant Nekrasov partition function in analogous way

Summary

$$SO(2N_f) \times U(1) + \text{S-duality} = E_{N_f+1}$$

Nekrasov partition function is invariant

Future work

$$N_f = 8?$$